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No metastable de Sitter vacua in $\mathcal{N} = 2$ supergravity with only hypermultiplets

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Abstract: We study the stability of vacua with spontaneously broken supersymmetry in $\mathcal{N} = 2$ supergravity theories with only hypermultiplets. Focusing on the projection of the scalar mass matrix along the sGoldstino directions, we are able to derive a universal upper bound on the lowest mass eigenvalue. This bound only depends on the gravitino mass and the cosmological constant, but not on the details of the quaternionic manifold spanned by the scalar fields. Comparing with the Breitenlohner-Freedman bound shows that metastability requires the cosmological constant to be smaller than a certain negative critical value. Therefore, only AdS vacua with a sufficiently negative cosmological constant can be stable, while Minkowski and dS vacua necessarily have a tachyonic direction.

Keywords: Supersymmetry Breaking, Extended Supersymmetry, Supergravity Models, dS vacua in string theory.

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Contents

1. Introduction

A crucial issue in string theory is to identify a mechanism for supersymmetry breaking which, at the same time, keeps the cosmological constant small, as current experimental observations suggest the existence of a tiny positive cosmological constant (dark energy) driving the expansion of the universe that we observe today. This has motivated the search for four-dimensional de Sitter (dS) vacua in string theory. One possible approach to this problem is to stay within the low-energy effective four-dimensional supergravity description and first determine the conditions under which a metastable vacuum exhibiting spontaneous supersymmetry breaking with a reasonably small cosmological constant can possibly arise. One may then similarly ask under which conditions it is possible to realize

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slow-roll inflation in such a setup. While finding the answers to these questions may not be sufficient for understanding the vacuum selection mechanism within string theory, it would certainly be a useful guideline for model building.

Arranging for metastable dS vacua in generic supersymmetric theories turns out to be surprisingly difficult. One of the reasons is that these vacua necessarily break supersymmetry spontaneously and hence supersymmetry does not guarantee the stability of the ground state. Actually, in refs. [1, 2] a necessary condition for the existence of metastable dS vacua within generic $\mathcal{N}=1$ supergravity theories was identified.[1] The crucial physical ingredient exploited in these analyses is the fact that in the scalar field space the most dangerous directions for metastability are the ones corresponding to the sGoldstinos, the supersymmetric partners of the Goldstino. While all the other multiplets can be made arbitrarily massive by suitably tuning the superpotential, the Goldstino multiplet is only allowed to have mass splittings induced by supersymmetry breaking. Thus the requirement for the sGoldstino square mass to satisfy the metastability bound (namely being positive in dS space and within the negative Breitenlohner-Freedman (BF) bound [4] in anti de Sitter (AdS) space) is independent of the superpotential but instead poses a strong necessary condition on the curvature of the scalar geometry. More precisely, along the sGoldstino direction the sectional curvature of the Kähler manifold spanned by the scalar fields has to have a limited size. Since the sGoldstino direction is determined by the superpotential this in turn poses also a constraint on the superpotential.

The aim of this paper is to pursue a similar study for $\mathcal{N}=2$ supergravity theories. The motivation for doing this is two-fold: Firstly, the scalar field space of $\mathcal{N}=2$ supergravity is not a special case of the $\mathcal{N}=1$ field space. Also, the scalar potential in $\mathcal{N}=2$ theories is fixed by a gauging of isometries, while the one of $\mathcal{N}=1$ theories is governed by an arbitrary superpotential. This makes the two analyses qualitatively different. Secondly, the hidden sector of string theory, where supersymmetry is believed to be spontaneously broken, often displays such an extended supersymmetry. Therefore an analysis in extended supergravity theories seems to be more suitable to establish the relation with higher-dimensional theories.

As a first step of this program we will focus in this paper on the simple situation of $\mathcal{N}=2$ theories which involve only hypermultiplets and a graviphoton gauging. As we will see in the body of the paper these theories are in some sense the analogs of $\mathcal{N}=1$ theories with only chiral multiplets.[2] The main result we find is that in $\mathcal{N}=2$ theories with only hypermultiplets metastability implies a negative upper bound on the cosmological constant and therefore dS vacua (as well as slow-roll inflation) are always excluded. A similar conclusion was also reached in the other particular situation of $\mathcal{N}=2$ theories involving only vector multiplets and Abelian gaugings, where metastability forces the cosmological constant to be negative [5, 6]. The study of more general situations, involving both hyper and vector multiplets and/or non-Abelian gaugings, is left to a subsequent paper [7]. In

[1] See also [3] for an analysis with similar spirit applied to the idea of landscape of vacua.

[2] However, they have the peculiarity of becoming trivial in the limit of rigid supersymmetry, where the graviphoton is decoupled. Indeed, in rigid $\mathcal{N}=2$ theories without vector multiplets the scalar potential vanishes identically.

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this more general type of theories a richer variety of possibilities is expected to exist. In fact some particular examples of stable dS vacua have already been constructed in this context, for instance in refs. [8, 9] exploiting non-Abelian gauge symmetries.

The paper is organized as follows. In section 2 we briefly review the results of refs. [1, 2] using a formalism that is tailored for the transition to $\mathcal{N}=2$ theories. In fact we slightly generalize the previous analyses in that we also derive a constraint for the existence of metastable AdS ground states with spontaneously broken supersymmetry. In section 3 we show that in $\mathcal{N}=2$ supergravities with only hypermultiplets no metastable dS ground states exist and derive a bound for the non-supersymmetric AdS vacua. Finally, in section 4 we summarize our conclusion and give an outlook on future directions of investigation. For completeness, we record the computations of the supertrace sum rules on boson and fermion masses for $\mathcal{N}=1$ and $\mathcal{N}=2$ theories in appendix A. We also summarize our conventions for the curvature of real and complex manifolds in appendix B.

2. N=1 theories with chiral multiplets

In order to prepare for the analysis in $\mathcal{N}=2$ supergravity, we shall start by briefly reviewing the conditions for the existence of metastable vacua in spontaneously broken $\mathcal{N}=1$ supergravity. We follow our earlier papers [1, 2] but use a slightly modified formalism, which makes the transition to $\mathcal{N}=2$ theories somewhat more suggestive. Furthermore, in [1, 2] we concentrated on finding dS vacua whereas in the following we extend the analysis to also include non-supersymmetric AdS vacua.

2.1 Preliminaries

Let us consider a generic $\mathcal{N}=1$ theory with n chiral multiplets Φ^i, containing complex scalar fields ϕ^i and chiral fermions χ^i [10]. This theory is described by a superpotential W and a Kähler potential K which defines a Kähler-Hodge geometry with a metric for the scalar fields given by $g_{i\bar{\jmath}}=K_{i\bar{\jmath}}$. The theory has a U(1) Kähler invariance which transforms $K \to K+f+\bar{f}$ and $W \to We^{-f}$. The holonomy of the scalar manifold is contained in $\mathrm{U}(1)\times\mathrm{U}(n)$, where the U(1) curvature form is identified with the Kähler form while the $\mathrm{U}(n)$ curvature is arbitrary.

Instead of choosing a Kähler gauge and describing the theory in terms of the invariant function $G=K+\log|W|^2$, we will use instead a different formulation where this symmetry is kept manifest. For this purpose, it is useful to introduce the quantity

$$L \equiv e^{K/2} W \,. \tag{2.1}$$

L transforms with weight $\frac{1}{2}$ under Kähler transformations: $L \to e^{-(f-\bar{f})/2}\,L$. It is then convenient to define covariant derivatives ∇ which include the U(1) Kähler connection in addition to the standard metric-compatible Christoffel connection. On a scalar quantity of weight p, for instance, one has $\nabla_i=\partial_i+pK_i$ and $\nabla_{\bar{\imath}}=\partial_{\bar{\imath}}-pK_{\bar{\imath}}$. The covariant derivatives of L are then found to be:

$$\nabla_{\bar{\imath}} L = 0\,, \qquad \nabla_i L = e^{K/2}\big(W_i+K_i W\big)\ . \tag{2.2}$$

From here one can see that L is covariantly anti-holomorphic with its holomorphic covariant derivative being related to the order parameters of supersymmetry breaking. Indeed the supersymmetry transformation of the fermions include the term $\delta_\epsilon \bar\chi^{\bar\imath} = -\sqrt{2}\,\bar\epsilon\, g^{\bar\imath j} N_j + \dots$, where the fermionic shifts N_i are given by:

$$N_i \equiv \nabla_i L \,. \tag{2.3}$$

For future reference let us also record the anti-holomorphic derivatives of N_i. These are simply given by

$$\nabla_{\bar\jmath} N_i = g_{i\bar\jmath}\, L \;\Rightarrow\; \nabla_i \bar N^j = \delta_i^j\, \bar L \,. \tag{2.4}$$

Also note that since ∇_i involve both the Christoffel connection and the U(1) Kähler connection, the commutator of two covariant derivatives acting on an object of non-zero U(1) weight has an additional piece coming from the U(1) curvature. For instance, on the fermionic shift one has:

$$\big[\nabla_i, \nabla_{\bar\jmath}\big] N_k = R_{i\bar\jmath k\bar s} \bar N^s - g_{i\bar\jmath} N_k \,, \tag{2.5}$$

where $R_{i\bar\jmath k\bar s}$ is the Riemann tensor of the Kähler manifold. (Our curvature conventions are summarized in appendix B.)

2.2 Mass matrices

Using the notation that we just introduced, the scalar potential V takes the following simple form:

$$V = \bar N^i N_i - 3|L|^2 \,. \tag{2.6}$$

Its first derivative is then given by:

$$\nabla_i V = -2 N_i \bar L + \bar N^j \nabla_i N_j \,. \tag{2.7}$$

Stationary points satisfy $\nabla_i V = 0$, and correspond to values of the scalar fields for which $\bar N^j\, \nabla_i N_j = 2 N_i \bar L$.

Let us now compute the bosonic and fermionic mass matrices at a generic stationary point. The scalar masses are given by the second derivatives of V. These are easily computed and can be partly simplified by using the identity (2.5). One finds:

$$\begin{aligned} \nabla_i \nabla_{\bar\jmath} V &= -2 g_{i\bar\jmath} |L|^2 + \nabla_i N_k \nabla_{\bar\jmath} \bar N^k - R_{i\bar\jmath p\bar q} N^p \bar N^{\bar q} + g_{i\bar\jmath} \bar N^k N_k - N_i \bar N_{\bar\jmath} \,, \\ \nabla_i \nabla_j V &= -\nabla_i N_j \bar L + \bar N^k \nabla_{(i} \nabla_{j)} N_k \,. \end{aligned} \tag{2.8}$$

The two independent blocks for the mass matrix are then given by:

$$m^2_{0 i\bar\jmath} = \nabla_i \nabla_{\bar\jmath} V \;, \qquad m^2_{0 ij} = \nabla_i \nabla_j V \,. \tag{2.9}$$

The fermionic mass matrix is also easy to compute. The mass terms for the physical fermions and the gravitino field can be read off from the following fermionic terms in the Lagrangian:

$$\begin{aligned} \mathcal{L}_{\rm fm} = &- L \psi^\mu \sigma_{\mu\nu} \psi^\nu - \bar L \bar\psi^\mu \bar\sigma_{\mu\nu} \bar\psi^\nu - \frac{i}{\sqrt{2}}\, N_i \chi^i \sigma_\mu \bar\psi^\mu + \frac{i}{\sqrt{2}}\, \bar N_{\bar\jmath} \bar\chi^{\bar\jmath} \bar\sigma_\mu \psi^\mu \\ &- \frac{1}{2} M_{ij} \chi^i \chi^j - \frac{1}{2} \bar M_{\bar\imath\bar\jmath} \bar\chi^{\bar\imath} \bar\chi^{\bar\jmath} + \dots \;, \end{aligned} \tag{2.10}$$

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where

$$M_{ij} \equiv \nabla_i N_j = \nabla_i \nabla_j L \,. \tag{2.11}$$

In the ground state the gravitino ψ^μ can be disentangled from the chiral fermions by the redefinition

$$\tilde{\psi}^\mu = \psi^\mu + \frac{i}{3\sqrt{2}}\, L^{-1} N_{\bar{\jmath}}\, \sigma^\mu \chi^{\bar{\jmath}} \,, \tag{2.12}$$

where L and $N_{\bar{\jmath}}$ are evaluated at the minimum of V. This results in the following mass matrices for the physical fields:

$$m_{3/2} = L \,, \qquad m_{1/2ij} = M_{ij} - \frac{2}{3} L^{-1} N_i N_j = \nabla_i N_j - \frac{2}{3} L^{-1} N_i N_j \,, \tag{2.13}$$

The mass matrices (2.9) and (2.13) obey a supersymmetric sum rule, which we record in appendix A.1.

2.3 Goldstino and sGoldstinos

As we already recalled, supersymmetry is spontaneously broken if in the vacuum $N_i \neq 0$. The associated Goldstino fermion is then given by the linear combination $\eta = N_i \chi^i$. This can be seen from the non-linear supersymmetry transformation of η and/or from the fact that in a Minkowski vacuum the vector N_i is a null vector of the physical mass matrix $m_{1/2ij}$. Indeed, from (2.11) and the stationarity condition following from (2.7) it is easy to see that $M_{ij} \bar{N}^j = 2\bar{L}\, N_i$. Using then (2.6) and (2.13) this implies

$$m_{1/2ij} \bar{N}^j = -\frac{2}{3} L^{-1} V N_i \,, \tag{2.14}$$

with the right hand side being zero when $V = 0$. The Goldstino field $\eta = N_i \chi^i$ has therefore a mass parameter which vanishes in Minkowski space and has a fixed value in units of the cosmological constant in AdS space:

$$m_\eta = -\frac{2}{3} m_{3/2}^{-1} V \,. \tag{2.15}$$

Finally the complex sGoldstino, i.e. the scalar field describing the supersymmetric partners of the Goldstino, is defined analogously by $\tilde{\eta} \equiv N_i \phi^i$.

2.4 Stability of supersymmetric vacua

Although in this paper we are interested in the stability of ground states with spontaneously broken supersymmetry, let us briefly present the proof that supersymmetric ground states are always stable. In this case one has $N_i = 0$, which automatically implies the stationarity condition coming from (2.7) and a semi-negative definite cosmological constant $V = -3\,|L|^2$. Moreover, the scalar mass matrix simplifies as follows:

$$m^2_{0i\bar{\jmath}} = \nabla_i N_k \nabla_{\bar{\jmath}} \bar{N}^k - 2 g_{i\bar{\jmath}} |L|^2 \,, \qquad m^2_{0ij} = -\bar{L} \nabla_i N_j \,. \tag{2.16}$$

Looking along an arbitrary direction $v^I = (v^i, v^{\bar{\imath}})$ in field space with normalization $v^I v_I = 1$ (or $v^i v_i = 1/2$), one finds:

$$\begin{aligned} m_0^2 &= m^2_{0I\bar{J}}\, v^I v^{\bar{J}} = 2\, m^2_{0i\bar{\jmath}}\, v^i v^{\bar{\jmath}} + m^2_{0ij}\, v^i v^j + m^2_{0\bar{\imath}\bar{\jmath}}\, v^{\bar{\imath}} v^{\bar{\jmath}} \\ &= \frac{1}{2}\big(2v^i \nabla_i N_k - v_k L\big)\big(2v^{\bar{\jmath}} \nabla_{\bar{\jmath}} \bar{N}^k - v^k \bar{L}\big) - \frac{9}{2}\, v^i v_i\, |L|^2 \,. \end{aligned} \tag{2.17}$$

In the last expression, the first term gives a semi-positive definite contribution so that m_0^2 satisfies the BF [4] bound[3]

$$m_0^2 \geq \frac{3}{4} V \,. \tag{2.18}$$

Notice that a minimal m_0^2 which saturates the bound can only be achieved along the special complex directions v_0^i for which the semi-positive terms are zero. These directions correspond to pseudo-eigenvectors of the matrix M_{ij}, in the sense that $M_i{}^{\bar{\jmath}} v_{0\,\bar{\jmath}} = 2L\, v_{0\,i}$.

2.5 Stability of non-supersymmetric vacua

Let us now turn to the stability of non-supersymmetric vacua, that is, those for which $N_i \neq 0$. This is largely discussed in refs. [1, 2] and here we only briefly recall the results. However we do extend our previous analysis in that we also include non-supersymmetric AdS ground states.

As was explained in detail in refs. [1, 2] the most stringent constraints on the stability of the ground state come from the directions of the two sGoldstinos. Therefore we focus on the sGoldstino subspace defined by the complex direction N_i and consider the quantity

$$m^2_{\tilde{\eta}} \equiv \frac{m^2_{0i\bar{\jmath}} \bar{N}^i N^{\bar{\jmath}}}{\bar{N}^k N_k} \,. \tag{2.19}$$

With the help of (2.6), (2.7) and (2.8) this can be rewritten as

$$m^2_{\tilde{\eta}} = R_{\tilde{\eta}} \bar{N}^i N_i + 2|L|^2 = 3\bigg(R_{\tilde{\eta}} + \frac{2}{3}\bigg)|m_{3/2}|^2 + R_{\tilde{\eta}}\, V \,, \tag{2.20}$$

where $R_{\tilde{\eta}}$ is the normalized holomorphic sectional curvature along the sGoldstino direction, namely

$$R_{\tilde{\eta}} = -\frac{R_{i\bar{\jmath}p\bar{q}} \bar{N}^i N^{\bar{\jmath}} \bar{N}^p N^{\bar{q}}}{(\bar{N}^k N_k)^2} \,. \tag{2.21}$$

The crucial observation is that $m^2_{\tilde{\eta}}$ represents an upper bound for the value of the smallest eigenvalue of the full mass matrix [1, 2].[4] Therefore a necessary condition for stability is that the value of $m^2_{\tilde{\eta}}$ should be non-negative for dS or Minkowski vacua and

[3] In AdS$_d$ the BF bound is given by $m^2 R^2 \geq -\frac{1}{4}(d-1)^2$, where R is the AdS radius. For $d = 4$ and $R^2 = -3V^{-1}$ this leads to the bound (2.18).

[4] In fact, the quantity $m^2_{\tilde{\eta}}$ arises as half of the trace of the two-dimensional submatrix of the full mass matrix along the two independent real directions that can be formed out of the complex Goldstino direction. It thus corresponds to the average of the two sGoldstino square masses. The splitting of these two masses depends explicitly on the superpotential and its derivatives, and is therefore less interesting.

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should satisfy the BF bound (2.18) for AdS vacua. It is convenient to phrase the discussion in terms of the following dimensionless parameter γ defined as

$$\gamma \equiv \frac{V}{3\,|m_{3/2}|^2}\,. \tag{2.22}$$

Minkowski/dS vacua correspond to $\gamma \in [0,+\infty)$ while AdS vacua have $\gamma \in [-1,0]$ since the cosmological constant is bounded to be larger than its critical supersymmetric value $V \geq -3|m_{3/2}|^2$. Stability requires $m^2_{\tilde{\eta}} \geq 0$ for dS vacua and $m^2_{\tilde{\eta}} \geq \frac{3}{4}V$ for AdS vacua which, using (2.20) and (2.22), can be viewed as the following bound for $R_{\tilde{\eta}}$:[5]

$$R_{\tilde{\eta}} \geq \begin{cases} -\dfrac{2}{3}\dfrac{1}{1+\gamma} & \text{for} \quad \gamma \geq 0\,, \\ -\dfrac{2}{3}\dfrac{1-\frac{9}{8}\gamma}{1+\gamma} & \text{for} \quad -1 \leq \gamma \leq 0\,. \end{cases} \tag{2.23}$$

From this expression we see that the condition for finding metastable vacua with broken supersymmetry becomes more and more restrictive as the cosmological constant is increased: AdS vacua with minimal cosmological constant ($\gamma \to -1$) can always exist, as in such a case the condition simply reads $R_{\tilde{\eta}} > -\infty$. On the other hand, Minkowski vacua ($\gamma = 0$) can exist only if $R_{\tilde{\eta}} \geq -\frac{2}{3}$. Finally, dS vacua with large cosmological constant ($\gamma \to +\infty$) can exist only if $R_{\tilde{\eta}} \geq 0$. The maximal freedom is therefore obtained in those models in which the sectional curvature $R_{\tilde{\eta}}$ either vanishes or turns out to be positive.

Notice finally that in the limit in which gravity is decoupled by sending the Planck scale to infinity while keeping the other scales fixed, the value of the quantity $m^2_{\tilde{\eta}}$ simplifies to the following expression:

$$m^2_{\tilde{\eta}} \simeq 3R_{\tilde{\eta}}(1+\gamma)|m_{3/2}|^2\,. \tag{2.24}$$

The standard limit of rigid supersymmetry can be obtained by further sending $m_{3/2}$ to zero. In that limit one finds $m^2_{\tilde{\eta}} \simeq R_{\tilde{\eta}}V$.

Another interesting thing to note is the fact that the product of several Kähler-Hodge manifolds is again a Kähler-Hodge manifold. Thanks to this property, it is actually easy to construct models satisfying the necessary condition (2.23). Indeed, starting with some manifolds $\mathcal{M}_i$ with sectional curvatures that are negative and bounded by some finite maximal values R_i, one can construct the product manifold $\mathcal{M} = \times_i \mathcal{M}_i$ and find directions along which the sectional curvature is still negative but larger (i.e. closer to zero) than any of the individual R_i, the maximal possible value being $R_{\rm min} = (\sum_i R_i^{-1})^{-1}$. This means in particular that, by taking sufficiently many copies of any given Kähler manifold, one can always satisfy the condition (2.23).

The fact that a Kähler manifold can factorize into several Kähler submanifolds also allows for situations in which the scalar fields spanning some of the submanifolds are

[5] Note that we use here and in (2.21) a different sign convention for the Ricci-, scalar- and sectional curvatures of Kähler manifolds compared to refs. [1, 2], although the Riemann tensor is defined in the same way. This is needed to consistently compare with the corresponding quantities for quaternionic-Kähler manifolds arising in next section. See appendix B for more details.

stabilized in a supersymmetric way whereas the scalar fields spanning the rest of the submanifolds spontaneously break supersymmetry, provided that the superpotential also has some special properties. For the non-supersymmetric sector, one would get again a condition like (2.23), where $R_{\tilde{\eta}}$ now refers to the relevant supersymmetry-breaking submanifold. For the supersymmetric sector, on the other hand, one should be careful as stability is not guaranteed in this case due to the fact that the cosmological constant is sourced by the other sector and departs from its critical supersymmetric value. As was shown in [11] for the particular case in which the two sectors interact only gravitationally, this cases cannot be viewed in general as a continuous limit of a supersymmetry breaking situation and therefore the stability of such vacua must then be studied separately.

3. $\mathcal{N}=2$ theories with hypermultiplets

So far we have reviewed the stability of non-supersymmetric ground states in $\mathcal{N}=1$ supergravity. Now we will move to the main topic of this paper and we will extend this analysis to the case of $\mathcal{N}=2$ supergravity coupled to an arbitrary number of hypermultiplets.

3.1 Preliminaries

Let us begin by reviewing the relevant aspects of the $\mathcal{N}=2$ theories and fix some conventions. For more details see, for example, refs. [12 – 15].[6] The gravitational multiplet contains the space-time metric $g_{\mu\nu}$, a pair of gravitini ψ_μ^A, $A=1,2$ and an Abelian graviphoton A_μ. This multiplet can be coupled to n hypermultiplets H^i, $i=1,\dots,n$ which contain $4n$ scalar fields q^u, $u=1,\dots,4n$ and $2n$ fermions ξ^α , $\alpha=1,\dots,2n$. The scalar fields q^u span a quaternionic-Kähler manifold of dimension $4n$ with holonomy group $\mathrm{Sp}(2n)\times\mathrm{SU}(2)$.

On a quaternionic-Kähler manifold there exists a triplet of almost complex structures J^x, $x=1,2,3$ which satisfy an SU(2) algebra. Associated with them is a triplet of Hyperkähler two-forms Ω^x which consequently obey

$$\Omega^x_{uw}\Omega^{yw}{}_v = -h_{uv}\delta^{xy} - \epsilon^{xyz}\Omega^z_{uv}\,, \tag{3.1}$$

where h_{uv} is the quaternionic metric. Furthermore, the Ω^x are identified with the field strength of the SU(2) part of the holonomy group and as a consequence they are covariantly constant with respect to the SU(2) connection: $\nabla_w\Omega^x_{uv}=0$.[7] Here and in the following, ∇_u denotes a covariant derivative involving also the SU(2) connection.

Our conventions are as follows. The SU(2) doublet indices A,B are raised and lowered in the usual way with the antisymmetric tensors ϵ_{AB} and ϵ^{AB} and the matrices $\sigma^x{}_A{}^B$ denote the usual antisymmetric Pauli matrices. The matrices σ^x_{AB} and σ^{xAB} are then symmetric

[6] In the following we discuss gauged $\mathcal{N}=2$ supergravity in the standard electric frame following refs. [12 – 15]. In principle it is also possible to gauge with respect to the magnetic graviphoton (see, for example, refs. [16]). However, if only the graviphoton is present, the symplectic rotation connecting the two cases is trivial and thus, without loss of generality, we can confine our discussion to the electric case.

[7] In fact Ω^x only needs to be proportional to the Hyperkähler two-forms, but the proportionality factor controls the normalization of the scalar kinetic terms and is thus important. We fix it to -1, as is usually done in the literature (corresponding to $\lambda=-1$ in [13, 14] and $\nu=-2$ in [15]).

and satisfy $(\sigma^x_{AB})^* = -\sigma^{xAB}$. They relate SU(2) triplets to the symmetric product of two SU(2) doublets, and can be used to alternatively describe any triplet as a bi-doublet through the definition $\xi^{AB} \equiv i\xi^x \sigma^{xAB}$. Moreover, they satisfy the identity:

$$\sigma^x_{AB}\sigma^x_{CD} = 2\epsilon_{A(C}\epsilon_{BD)} \,. \tag{3.2}$$

For the Sp$(2n)$ group, we denote by $\alpha, \beta = 1, \ldots, 2n$ the $2n$-plets indices. These are raised and lowered with the antisymmetric symplectic tensors $C_{\alpha\beta}$ and $C^{\alpha\beta}$.

It is convenient to define a vielbein $U_u^{\alpha A}$ for the quaternionic metric by the relation $h_{uv} = U_u^{\alpha A} U_v^{\beta B} \epsilon_{AB} C_{\alpha\beta}$. The inverse vielbein $U^u_{\alpha A}$ then satisfies $U^u_{\alpha A} U_v^{\alpha A} = \delta^u_v$ and $U_u^{\alpha A} U^u_{\beta B} = \delta^\alpha_\beta \delta^A_B$. These actually satisfy the stronger relations $h_{uv} = \epsilon_{AB} U_u^{\alpha A} U^B_{v\alpha}$ and $\Omega^x_{uv} = -i\sigma^x_{AB} U_u^{\alpha A} U^B_{v\alpha}$, or $U^u_{\alpha A} U^v_{\beta B} h_{uv} = \epsilon_{AB} C_{\alpha\beta}$ and $U^u_{\alpha A} U^v_{\beta B} \Omega^x_{uv} = -i\sigma^x_{AB} C_{\alpha\beta}$, which are conveniently summarized in the identity:

$$U_u^{\alpha A} U^B_{\alpha v} = \frac{1}{2} h_{uv} \epsilon^{AB} - \frac{i}{2} \Omega^x_{uv} \sigma^{xAB} \,. \tag{3.3}$$

The curvature consists of an SU(2) part and an Sp$(2n)$ part with the corresponding curvature forms given by:

$$R^{AB}_{uv} = -i\,\Omega^x_{uv}\sigma^{xAB} \,, \qquad R^{\alpha\beta}_{uv} = \epsilon_{AB} U^{\gamma A}_{[u} U^{\delta B}_{v]} \big(-2\,\delta^\alpha_{(\gamma}\delta^\beta_{\delta)} + \Sigma^{\alpha\beta}{}_{\gamma\delta}\big) \,. \tag{3.4}$$

The tensor $\Sigma_{\alpha\beta\gamma\delta}$ must be completely symmetric but is otherwise arbitrary, and represents the only freedom in the curvature. The full Riemann tensor with two 'flat' index-pairs is given by $R^{\alpha A\beta B}{}_{uv} = R^{AB}_{uv} C^{\alpha\beta} + R^{\alpha\beta}_{uv}\epsilon^{AB}$. Using eq. (3.2) the curvature with only flat indices is found to be

$$R_{\alpha A\beta B\gamma C\delta D} = 2\,\epsilon_{A(C}\epsilon_{BD)} C_{\alpha\beta} C_{\gamma\delta} + \epsilon_{AB}\epsilon_{CD}\big(-2\,C_{\alpha(\gamma}C_{\beta\delta)} + \Sigma_{\alpha\beta\gamma\delta}\big) \,. \tag{3.5}$$

Its version with only curved indices is instead given by:

$$R_{uvrs} = -h_{u[r}h_{vs]} - \Omega^x_{uv}\Omega^x_{rs} - \Omega^x_{u[r}\Omega^x_{vs]} + \Sigma_{uvrs} \,, \tag{3.6}$$

where:

$$\Sigma_{uvrs} = \epsilon_{AB}\epsilon_{CD} U_u^{\alpha A} U_v^{\beta B} U_r^{\gamma C} U_s^{\delta D} \Sigma_{\alpha\beta\gamma\delta} \,. \tag{3.7}$$

The tensor Σ_{uvrs} behaves like a Weyl component of the Riemann tensor, in the sense that any contraction with the metric vanishes. This implies that the Ricci tensor is completely universal and that quaternionic-Kähler manifolds are Einstein manifolds with

$$R_{uv} = -2(n+2)h_{uv} \,, \qquad R = -8n(n+2) \,. \tag{3.8}$$

So far we have discussed the ungauged $\mathcal{N} = 2$ theory. Let us now turn to the situation in which an isometry of h_{uv} is gauged with the graviphoton A_μ. In this case the scalars are charged under the isometry group and transform as $\delta q^u = \Lambda\, k^u(q)$, where Λ is the space-time dependent gauge parameter while $k^u(q)$ is the Killing vector, which satisfies the Killing equation

$$\nabla_{(u}k_{v)} = 0 \,. \tag{3.9}$$

In the Lagrangian all the space-time derivatives acting on scalar fields are then replaced by covariant derivatives, of the form $D_\mu q^u \equiv \partial_\mu q^u + k^u A_\mu$.

On a quaternionic-Kähler manifold, any Killing vector k^u can be expressed in terms of a triplet of Killing potentials P^x, defined by

$$\nabla_u P^x = 2\,\Omega^x_{uv} k^v\,. \tag{3.10}$$

Actually one can also relate k^u and P^x as:

$$k_u = -\frac{1}{6}\,\Omega^x_{uv}\nabla^v P^x\,, \qquad P^x = \frac{1}{2n}\,\Omega^x_{uv}\nabla^u k^v\,. \tag{3.11}$$

One also finds the following relations for the second derivatives of these quantities:

$$\begin{aligned} \big[\nabla_u,\nabla_v\big] P^x &= 2\,\epsilon^{xyz}\Omega^y_{uv}P^z\,,\\ \big[\nabla_u,\nabla_v\big] k_w &= R_{uvws}k^s\,,\\ \nabla_u\nabla_v k_w &= -R_{vwus}k^s\,. \end{aligned} \tag{3.12}$$

Moreover, P^x and k_u satisfy the harmonic equations

$$\nabla^w\nabla_w P^x = 4nP^x\,, \qquad \nabla^w\nabla_w k_u = 2(n+2)\,k_u\,. \tag{3.13}$$

Finally, the derivatives of the Killing potentials P^x turn out to be related to the order parameters of supersymmetry breaking. Indeed, the supersymmetry transformation of the hyperini has the form $\delta\xi_\alpha = N^A_\alpha\epsilon_A + \dots$, where the fermionic shifts N^A_α are given by:

$$N^A_\alpha = 2U^A_{u\alpha}k^u = \frac{1}{3}U^u_{\alpha B}\nabla_u P^{AB}\,. \tag{3.14}$$

3.2 Mass matrices

The scalar potential can be expressed in terms of the Killing vector and the Killing potentials, and takes the following simple form:

$$V = N^\alpha_A N^A_\alpha - 3\,P^xP^x = 4\,k^w k_w - 3\,P^xP^x\,. \tag{3.15}$$

Its first derivatives are given by

$$\nabla_u V = 8\,k^w\nabla_u k_w - 6\,P^x\nabla_u P^x\,, \tag{3.16}$$

and stationary points where $\nabla_u V = 0$ are thus characterized by the condition $k^w\nabla_u k_w = \frac{3}{4}P^x\nabla_u P^x$.

The scalar mass matrix at a stationary point of the potential is related to the second derivatives of the potential. These are found to be:

$$\nabla_u\nabla_v V = 8\,\nabla_u k^w\nabla_v k_w - 8\,R_{usvt}k^sk^t - 6\,\nabla_u P^x\nabla_v P^x - 6\,P^x\nabla_{(u}\nabla_{v)}P^x\,. \tag{3.17}$$

In the conventions we are following, the kinetic term of the scalar fields has the non-canonical form $\mathcal{L}_{\rm kin} = -h_{uv}D_\mu q^u D^\mu q^v$. The properly normalized mass matrix for the scalars is thus given by:

$$m^2_{0uv} = \frac{1}{2}\nabla_u\nabla_v V\,. \tag{3.18}$$

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The square mass of the graviphoton is induced by the connection terms in the covariant derivatives of the scalars kinetic term. Taking into account that with the conventions we are following the kinetic term for the graviphoton has the non-canonical form $\mathcal{L}_{\rm kin} = -\frac{1}{8}F_{\mu\nu}F^{\mu\nu}$, one deduces that:

$$m_1^2 = 4k^u k_u\,. \tag{3.19}$$

The mass terms for the hyperini and the gravitini can be read off from the fermionic part of the $\mathcal{N}=2$ Lagrangian

$$\begin{aligned}\mathcal{L}_{\rm fm} = {} & P_{AB}\bar{\psi}^A_\mu\gamma^{\mu\nu}\psi^B_\nu + \bar{P}^{AB}\bar{\psi}_{A\mu}\gamma^{\mu\nu}\psi_{\nu B} + 2iN^A_\alpha\bar{\xi}^\alpha\gamma_\mu\psi^\mu_A + 2iN^\alpha_A\bar{\xi}_\alpha\gamma_\mu\psi^{\mu A} \\ & + M_{\alpha\beta}\bar{\xi}^\alpha\xi^\beta + \bar{M}^{\alpha\beta}\bar{\xi}_\alpha\xi_\beta + \dots\,,\end{aligned} \tag{3.20}$$

where

$$M_{\alpha\beta} = -U^u_{\alpha A}U^v_{\beta B}\epsilon^{AB}\nabla_{[u}k_{v]} = -\frac{1}{6}\,U^u_{\alpha A}U^v_{\beta B}\nabla_u\nabla_v P^{AB}\,. \tag{3.21}$$

In order to disentangle the gravitino from the Goldstino, one redefines

$$\tilde{\psi}^{\mu A} = \psi^{\mu A} + \frac{i}{3}P^{-1AB}N^\beta_B\gamma^\mu\xi_\beta\,, \tag{3.22}$$

which results in the following mass matrices for the physical fermions and the two gravitini[8]

$$\begin{aligned} m_{1/2\alpha\beta} &= M_{\alpha\beta} - \frac{4}{3}\bar{P}^{-1}_{AB}\,N^A_\alpha N^B_\beta = -U^u_{\alpha A}U^v_{\beta B}\left(\epsilon^{AB}\nabla_{[u}k_{v]} + \frac{16}{3}P^{AB}|m_{3/2}|^{-2}k_u k_v\right), \\ m_{3/2AB} &= P_{AB}\,. \end{aligned} \tag{3.23}$$

Thus, the gravitino mass scale is simply given by:

$$|m_{3/2}| = \sqrt{P^x P^x}\,. \tag{3.24}$$

Comparing with the formulation of $\mathcal{N}=1$ theories described in section 2, we can now identify the generalization of each ingredient to the $\mathcal{N}=2$ case. We see that P^x is the generalization of L while N^A_α is instead the generalization of N_i.

3.3 Goldstinos and sGoldstinos

Supersymmetry is spontaneously broken whenever $N^A_\alpha \neq 0$ on the vacuum. The corresponding two Goldstino fermions are then given by $\eta^A = N^A_\alpha\xi^\alpha$. Using the stationarity condition following from (3.16) and the properties of the vielbein one can show that

$$M_{\alpha\beta}N^\beta_A = 2\,P_{AB}N^B_\alpha\,. \tag{3.25}$$

Using (3.23) together with the relation $N^\alpha_A N^B_\alpha = 2\,k^w k_w\,\delta^B_A$, eq. (3.25) implies

$$m_{1/2\alpha\beta}\,N^\beta_A = -\frac{2}{3}\,V\bar{P}^{-1}_{AB}N^B_\alpha\,. \tag{3.26}$$

[8]Notice that $P_{AC}\bar{P}^{CB} = P^x P^x\,\delta^B_A$. It follows that $P^{-1AB} = (P^x P^x)^{-1}\bar{P}^{AB}$ and similarly $\bar{P}^{-1}_{AB} = (P^x P^x)^{-1}P_{AB}$.

Thus we see again that the normalized mass matrix for the two Goldstinos vanishes identically in Minkowski space and has a fixed form in units of the cosmological constant in AdS space:

$$m_{\eta AB} = -\frac{2}{3} m^{-1}_{3/2AB} V \,. \tag{3.27}$$

The two independent Goldstino fermions $\eta^A = N^A_\alpha \xi^\alpha$, which transform as a doublet under SU(2), have four real sGoldstino partners given by $\tilde{\eta}^{AB} = N^{AB}_u q^u$. The quantity N^{AB}_u transforms as the tensor product of two SU(2) doublets, and can be computed by acting with the inverse vielbein $U^{\alpha A}_u$ on N^B_α. This is a result of the fact that $U^{\alpha A}_u$ locally maps the tangent space where the fermions are defined to the coordinates of the manifold associated with the scalar fields. More precisely, one finds:

$$N^{AB}_u = U^{\alpha A}_u N^B_\alpha = N_u \, \epsilon^{AB} + i N^x_u \, \sigma^{xAB} \,, \tag{3.28}$$

where in the second equation we used the identity (3.3) to decompose N^{AB}_u into a singlet N_u plus a triplet N^x_u with

$$N_u = k_u \,, \qquad N^x_u = -\Omega^{xv}_u k_v = -\frac{1}{2} \nabla_u P^x \,. \tag{3.29}$$

The four-dimensional space of sGoldstino directions can thus be parameterized by (N_u, N^x_u).[9] These vectors form an orthonormal basis, in the sense that:

$$N^u N_u = k^u k_u \,, \qquad N^{xu} N^y_u = k^u k_u \, \delta^{xy} \,, \qquad N^u N^x_u = 0 \,. \tag{3.30}$$

It is then convenient to use the fields $\tilde{\eta} = N_u q^u$ and $\tilde{\eta}^x = N^x_u q^u$ to parameterize the four independent sGoldstinos.

3.4 Stability of supersymmetric vacua

Let us consider first the case of supersymmetric vacua. Unbroken supersymmetry implies

$$k_u = 0 \;\Rightarrow\; N_u = N^x_u = 0 \,. \tag{3.31}$$

As usual, any point in the scalar field space where these conditions are fulfilled is automatically a stationary point of the potential, as can be seen from eq. (3.16). At such points the cosmological constant is negative and given by $V = -3 P^x P^x$. Moreover, the mass matrix (3.18) simplifies and can be rewritten as

$$\begin{aligned} m^2_{0uv} &= 4 \, \nabla_u k^w \nabla_v k_w - 3 \, P^x \nabla_{(u} \nabla_{v)} P^x \\ &= 4 \Big(\nabla_u k^w - \frac{3}{4} P^x \Omega^{xw}_u \Big) \Big(\nabla_v k_w - \frac{3}{4} P^y \Omega^y_{vw} \Big) - \frac{9}{4} \, h_{uv} P^x P^x \,. \end{aligned} \tag{3.32}$$

In the last expression, the first term is semi-positive definite, so the value of the mass matrix along any normalized direction v^u, with $v^u v_u = 1$, satisfies the BF bound (2.18) which guarantees stability: $m^2_0 = m^2_{0uv} v^u v^v \geq \frac{3}{4} V$. Note that the directions v^u_0 in field space for which this bound is saturated satisfy an equation of the form $(\nabla_u k_v) v^v_0 = \frac{3}{4} \, P^x \Omega^x_{uv} v^v_0$.

[9]Note that N^x_u conjugates N_u with respect to each of the three almost complex structures Ω^{xv}_u.

3.5 Stability of non-supersymmetric vacua

Let us now study the conditions under which metastable non-supersymmetric vacua can exist. Spontaneously broken supersymmetry implies

$$k_u \neq 0 \;\Rightarrow\; N_u, N_u^x \neq 0\,. \tag{3.33}$$

One can then study the mass matrix in the four-dimensional subspace of sGoldstino directions spanned by the four vectors $N_u = k_u$ and $N_u^x = -\Omega_u^{xv} k_v$. Gauge invariance of the potential implies however that at any stationary point the vector N_u is a flat direction of the potential, corresponding to the would-be Goldstone boson that is eaten by the graviphoton. Let us then study the mass matrix in the three-dimensional subspace defined by the vectors N_u^x given by

$$m_{\tilde\eta}^{2xy} = \frac{m^2_{0uv} N^{ux} N^{vy}}{N^w N_w}\;. \tag{3.34}$$

This expression for $m_{\tilde\eta}^{2xy}$ can be simplified using equations (3.17) and (3.18) and the stationarity condition coming from (3.16).[10] One then finds, after a straightforward computation, the following simple expression

$$m_{\tilde\eta}^{2xy} = -4\big(R_{\tilde\eta}^{xy} + 3\,\delta^{xy}\big) k^w k_w + 4\big(\delta^{xy} - \pi^{xy}\big) P^z P^z\,, \tag{3.35}$$

where

$$\pi^{xy} = \frac{P^x P^y}{P^z P^z} \tag{3.36}$$

is the projector along the direction defined by P^x and $R_{\tilde\eta}^{xy}$ is given by

$$R_{\tilde\eta}^{xy} = \frac{R_{usvt} N^{ux} N^s N^{vy} N^t}{(N^w N_w)^2}\;. \tag{3.37}$$

This quantity is something like a tri-holomorphic sectional curvature for the quaternionic directions N_u^{AB}, in the sense that its diagonal elements correspond to the three independent holomorphic sectional curvatures that one can build out of N_u and one of its conjugates $N_u^x = J^x_{uv} N^v$. Using the expression (3.6) for the Riemann tensor, one can evaluate $R_{\tilde\eta}^{xy}$ more explicitly, and express it in terms of the tensor $\Sigma_{\alpha\beta\gamma\delta}$. One actually finds:

$$R_{\tilde\eta}^{xy} = -2\,\delta^{xy} - \frac{\Sigma_{\alpha\beta\gamma\delta} N^{\alpha A} N^{\beta B} N^{\gamma C} N^{\delta D}}{(N^{\epsilon E} N_{\epsilon E})^2}\;\sigma^x_{AB}\sigma^y_{CD}\,. \tag{3.38}$$

Metastability of the vacuum requires that the eigenvalues of the three-dimensional matrix $m_{\tilde\eta}^{2xy}$ given in (3.35) should be either positive or above the BF bound, depending on the sign of the cosmological constant. This condition depends on the tensor $\Sigma_{\alpha\beta\gamma\delta}$ in a non-trivial way, and can be understood as a constraint on it. More precisely, it restricts the values that the curvature is allowed to take in the subspace of sGoldstino directions.

[10] The main intermediated step needed is the relation $\nabla^w k_u \nabla_w P^x = 3 P^x k_u + \frac{1}{2}\epsilon^{xyz} P^y \nabla_u P^z$. This can be derived by taking a derivative of the identity $k^w \nabla_w P^x = 0$ and using then the stationarity condition and the first relation in (3.12).

As in the previous section, to analyze the implications of the metastability constraints it is convenient to parametrize the value of the cosmological constant through the dimensionless parameter $\gamma = V/(3|m_{3/2}|^2)$.

To study the matrix $m^{2xy}_{\tilde\eta}$, it is convenient to switch to a basis of eigenvectors of the projector π^{xy}, which we shall denote by v^x_i, $i = 1, 2, 3$, for the eigenvalues $\lambda_i = (1, 0, 0)$ (so that v^x_1 is the direction defined by P^x and $v^x_{2,3}$ span the subspace orthogonal to it). These vectors can be chosen in such a way as to form an orthonormal and complete basis of the three-dimensional space under consideration, with:

$$\begin{aligned} &\pi^{xy} v^y_i = \lambda_i v^x_i \quad \text{(no sum on } i\text{)} \quad \text{and} \quad \lambda_i = (1,0,0)\,, \\ &v^x_i v^x_j = \delta_{ij}\,, \qquad v^x_i v^y_i = \delta^{xy}\,. \end{aligned} \tag{3.39}$$

In this new basis, the matrix $m^2_{\tilde\eta ij} \equiv m^{2xy}_{\tilde\eta} v^x_i v^y_j$ is still not diagonal. But each of its diagonal elements must nevertheless necessarily satisfy the metastability bound on their own. These three elements define indeed the values of the square mass along the three special orthogonal directions $v^x_i N^x_u$, which we shall denote by:

$$m^2_{\tilde\eta i} \equiv m^{2xy}_{\tilde\eta} v^x_i v^y_i \quad \text{(no sum on } i\text{)}\,. \tag{3.40}$$

Using (3.35) and (3.39) one computes

$$m^2_{\tilde\eta i} = -3\Big(R_{\tilde\eta i} + \frac{5}{3} + \frac{4}{3}\lambda_i\Big)|m_{3/2}|^2 - \Big(R_{\tilde\eta i} + 3\Big)V\,, \tag{3.41}$$

in terms of the holomorphic sectional curvatures defined by the rotated complex structures $J_{iuv} = J^x_{uv} v^x_i$, which are given by:

$$R_{\tilde\eta i} \equiv R^{xy}_{\tilde\eta} v^x_i v^y_i \quad \text{(no sum on } i\text{)}\,. \tag{3.42}$$

The metastability condition ($m^2_0 \geq 0$ if $V \geq 0$ and $m^2_0 \geq \frac{3}{4}V$ if $V < 0$) applied to $m^2_{\tilde\eta i}$ then implies

$$R_{\tilde\eta i} \leq \begin{cases} -\dfrac{5{+}4\lambda_i}{3}\,\dfrac{1{+}\frac{9}{5+4\lambda_i}\gamma}{1+\gamma}\,, & \gamma \geq 0\,, \\ -\dfrac{5{+}4\lambda_i}{3}\,\dfrac{1{+}\frac{45}{4(5+4\lambda_i)}\gamma}{1+\gamma}\,, & \gamma \leq 0\,. \end{cases} \tag{3.43}$$

Summarizing, we see that in $\mathcal{N} = 2$ theories we get three conditions, all similar to the one of $\mathcal{N} = 1$. They are associated with three of the partners of the two independent Goldstinos. Note however that the coefficients in the quantities $m^2_{\tilde\eta i}$ differ from the coefficients in the $\mathcal{N} = 1$ quantity $m^2_{\tilde\eta}$ given in (2.20). This is reasonable, since the geometry is quaternionic-Kähler for $\mathcal{N} = 2$ and Kähler-Hodge for $\mathcal{N} = 1$, and these two kinds of geometries are unrelated.[11] Furthermore, note that the sectional curvatures enter (2.20) and (3.41) with a different sign, once compatible conventions for real and complex manifolds

[11]A notable exception to this general fact is given by the family of coset manifolds $SU(2,n)/(U(1) \times SU(2) \times SU(n))$, which turn out to be both Kähler-Hodge and quaternionic-Kähler.

are used (see appendix B). This results in opposite inequality signs in the metastability constraints on the sectional curvature given in (2.23) and (3.43).

Before we proceed let us inspect the limit where gravity is decoupled by sending the Planck scale to infinity. In this limit, the $\mathcal{N}=2$ geometry becomes Hyperkähler while the $\mathcal{N}=1$ geometry becomes Kähler. The two geometries are then related, in the sense that the former is just a subclass of the latter. As a result, $\mathcal{N}=2$ theories reduce to a special case of $\mathcal{N}=1$ theories, and the metastability conditions arising in the two cases can be directly compared. In this rigid limit, however, in which the graviton, the gravitino and the graviphoton are decoupled, the scalar potential of $\mathcal{N}=2$ theories with only hypermultiplets becomes trivial. This corresponds to the fact that from the $\mathcal{N}=1$ perspective the superpotential vanishes. As a result also the sGoldstino masses go to zero, independently of the curvature of the Hyperkähler manifold and we have

$$m^2_{\tilde{\eta} i} \simeq 0\,. \tag{3.44}$$

This means that in this limit the $\mathcal{N}=2$ conditions can never really be satisfied, since the potential identically vanishes and thus the scalar fields cannot be stabilized. The $\mathcal{N}=1$ conditions implied by (2.24), on the other hand, can be satisfied for models with suitable geometry, but when the superpotential is sent to zero the scalar masses flow to zero also in this case.

Up to now we have not used the fact that quaternionic-Kähler manifolds have a constrained curvature tensor with a sectional curvature given in (3.38). Similarly, the $R_{\tilde{\eta} i}$ that appear in (3.41) take the form:

$$R_{\tilde{\eta} i} = -2 + \Delta_i(\Sigma)\,, \tag{3.45}$$

where

$$\Delta_i(\Sigma) \;\equiv\; \frac{\Sigma_{\alpha\beta\gamma\delta} N^{\alpha A} N^{\beta B} N^{\gamma C} N^{\delta D}}{(N^{\epsilon E} N_{\epsilon E})^2}\, v_i^x \sigma^x_{AB} v_i^y \sigma^y_{CD} \quad \text{(no sum on } i) \;. \tag{3.46}$$

So the metastability conditions constrain the allowed values for the quantities $\Delta_i(\Sigma)$, for a given value of the parameter γ.

As a first remark, note that for those particular quaternionic-Kähler manifolds for which the tensor $\Sigma_{\alpha\beta\gamma\delta}$ vanishes, the situation simplifies substantially.[12] Indeed, in that case one simply has $R_{\tilde{\eta} i} = -2$, and thus $m^2_{\tilde{\eta} i} = (1-4\lambda_i)|m_{3/2}|^2 - V$, that is $m^2_{\tilde{\eta}1} = -V - 3\,|m_{3/2}|^2$ in the direction parallel to P^x and $m^2_{\tilde{\eta}2,3} = -V + |m_{3/2}|^2$ along the two directions orthogonal to P^x. These satisfy the stability bound only if $\gamma \in [-1, -\frac{4}{7}]$, and thus Minkowski/dS vacua are excluded.

Even for more general quaternionic-Kähler manifolds with $\Sigma \neq 0$, we can actually obtain a stronger constraint from (3.41). Notice in this respect that the three square masses (3.41) transform as a triplet under SU(2) R-symmetry transformations, reflecting the fact that they are associated with the triplet of almost complex structures existing on quaternionic-Kähler manifolds. One may then try to look for an SU(2) singlet projection

[12]These correspond to the family of coset manifolds $\mathrm{Sp}(2,2n)/(\mathrm{Sp}(2)\times\mathrm{Sp}(2n))$.

and check whether it leads to any useful information. From the point of view of the original mass matrix $m^{2xy}_{\tilde\eta}$, the only object that could lead to such a thing is the trace. More precisely, one can consider the average of the diagonal elements, which by the completeness relation in (3.39) also corresponds to the average of the three masses $m^2_{\tilde\eta i}$ computed above:

$$m^2_{\tilde\eta} \equiv \frac{1}{3}\delta^{xy} m^{2xy}_{\tilde\eta} = \frac{1}{3}\textstyle\sum_i m^2_{\tilde\eta i}\,. \tag{3.47}$$

Using (3.35) one arrives at

$$m^2_{\tilde\eta} = -3\Big(R_{\tilde\eta} + \frac{19}{9}\Big)|m_{3/2}|^2 - \Big(R_{\tilde\eta} + 3\Big)V\,, \tag{3.48}$$

where $R_{\tilde\eta}$ is the averaged sectional curvature

$$R_{\tilde\eta} \equiv \frac{1}{3}\,\delta^{xy} R^{xy} = \frac{1}{3}\textstyle\sum_i R_{\tilde\eta i}\,. \tag{3.49}$$

Note now that $m^2_{\tilde\eta}$ also gives an upper bound on the smallest mass eigenvalue, as a consequence of the fact that each $m^2_{\tilde\eta i}$ gives itself a lower bound.[13] The metastability condition applied to $m^2_{\tilde\eta}$ then implies

$$R_{\tilde\eta} \leq \begin{cases} -\dfrac{19}{9}\,\dfrac{1+\frac{27}{19}\,\gamma}{1+\gamma}\;, & \gamma \geq 0\,, \\[2ex] -\dfrac{19}{9}\,\dfrac{1+\frac{135}{76}\,\gamma}{1+\gamma}\,, & \gamma \leq 0\,. \end{cases} \tag{3.50}$$

The crucial observation that one can make at this point is that the averaged sectional curvature $R_{\tilde\eta}$ actually is independent of the tensor $\Sigma_{\alpha\beta\gamma\delta}$ and thus takes a universal value common to all the possible quaternionic-Kähler manifolds. Indeed, using the property (3.2) in (3.45) and (3.49), one finds:[14]

$$R_{\tilde\eta} = -2 - \frac{2}{3}\,\frac{\Sigma_{\alpha\beta\gamma\delta} N^{\alpha A} N^{\beta B} N^{\gamma C} N^{\delta D}}{(N^{\epsilon E} N_{\epsilon E})^2}\;\epsilon_{AB}\epsilon_{CD} = -2\,. \tag{3.51}$$

Inserting (3.51) into (3.48) one then finds the simple expression

$$m^2_{\tilde\eta} = -\frac{1}{3}\big(1 + 9\,\gamma\big)|m_{3/2}|^2\,. \tag{3.52}$$

This satisfies the metastability bound only for

$$\gamma \in \big[-1, -\frac{4}{63}\big]\,. \tag{3.53}$$

[13]Indeed, $m^2_{\tilde\eta}$ is the averaged trace of the matrix, and gives thus the average of the eigenvalues. Each $m^2_{\tilde\eta i}$ is instead just the projection of the matrix along a specific direction, and is thus a combination of the eigenvalues with coefficients whose square sum up to 1. In both cases, the resulting value is clearly an upper bound to the smallest eigenvalue of $m^{2xy}_{\tilde\eta}$, and thus also of the full mass matrix m^2_{0uv}.

[14]This follows from the fact that the contraction $N^{\alpha A} N^{\beta B}\epsilon_{AB}$ is antisymmetric in α, β whereas the tensor $\Sigma_{\alpha\beta\gamma\delta}$ is completely symmetric in all indices.

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Notice that this restriction implies in paticular that dS vacua are always excluded.[15] This is unavoidable and holds true for any scalar geometry.[16] AdS vacua, on the other hand, are allowed if they satisfy (3.53). This represents the main result of our investigation.

Notice finally that the product of several quaternionic-Kähler manifolds is no longer a quaternionic-Kähler manifold. This is a consequence of the particular form that the Riemann curvature tensor must take. More precisely, the Ricci- and scalar curvatures are completely fixed by the dimensionality of the space (c.f. (3.8)), and this relation is destroyed when taking the product of two of such manifolds. Thus, there is no easy way of diluting the curvature just by taking products of manifolds and thus the bound is always unavoidably violated.

4. Conclusions and outlook

In this paper we have performed a general study on the conditions under which locally stable vacua with spontaneously broken supersymmetry can occur in $\mathcal{N}=2$ supergravity theories with only hypermultiplets. The results have been compared with the corresponding conditions that were already known for $\mathcal{N}=1$ supergravity theories with only chiral multiplets [1, 2]. As in the $\mathcal{N}=1$ case, our strategy has been to look at the most dangerous scalar fluctuations, which are the ones related to the scalar partners of the Goldstino fermion, the sGoldstinos.

In the $\mathcal{N}=1$ case the constraint can be formulated as a lower bound on the curvature of the scalar manifold spanned by the scalar components of the chiral multiplets. More concretely, they represent a lower bound on the holomorphic sectional curvature in the complex sGoldstino direction defined by the complex structure of the Kähler-Hodge scalar manifold. They constrain therefore both the allowed scalar geometries and the allowed supersymmetry breaking directions. In the $\mathcal{N}=2$ case, we have found that there are three constraints on the curvature of the scalar manifold, which are all similar to the one arising in $\mathcal{N}=1$ theories. This corresponds to the fact that in this case there are more sGoldstinos. More precisely, one finds an upper bound on the three possible holomorphic sectional curvatures in the complex sGoldstino directions defined by the three almost complex structures of the quaternionic-Kähler scalar manifold. However, it turns out that the quaternionic-Kähler geometry underlying $\mathcal{N}=2$ models implies a very restricted form of the curvature tensor, which is completely fixed up to a Weyl-type contribution Σ. This is in contrast with the Kähler-Hodge geometry underlying $\mathcal{N}=1$ theories, which allows instead for a generic curvature tensor. As a consequence, the average of the three holomorphic sectional curvatures arising in the $\mathcal{N}=2$ constraints happens to have a fixed constant value

[15]This result was already know to hold for the particular subclasses of quaternionic-Kähler manifolds for which $n=1$ as well as those with $n>1$ and $\Sigma_{\alpha\beta\gamma\delta}=0$ [17].

[16]There is an apparent counter-example of this result in ref. [18], where a metastable dS vacuum was found in the universal hypermultiplet geometry with instanton corrections taken into account. However the approximation used does not keep the metric quaternionic and we suspect that the dS vacuum is destabilized once the higher instanton corrections required to make the metric quaternionic are included. We understand that preliminary investigations point in this direction and we thank F. Saueressig for discussions on this issue.

independent of Σ, which translates into a universal negative upper bound on the values of the cosmological constant that are compatible with the metastability of the vacuum. This implies in particular that metastable dS vacua are excluded, independently of the specific scalar geometry of the model.[17]

The strong results that we find for $\mathcal{N}=2$ theories in the case with only hypermultiplets are very similar to the comparably strong results holding in the case in which only vector multiplets are present and the gauging is Abelian [5, 6]. They both have to do with the restricted form that the curvature of the scalar manifolds, which are respectively quaternionic-Kähler and special-Kähler, is allowed to take. In fact, the upper bounds on the lowest mass eigenvalue in these two special cases read

$$\begin{aligned} m^2_{\text{hyper}} &\le -V - \frac{1}{3}|m_{3/2}|^2\,, \\ m^2_{\text{vector}} &\le -2V\,. \end{aligned} \tag{4.1}$$

Similar tachyonic modes seem to be endemic also in $\mathcal{N} > 2$ theories; see for instance refs. [20].

Another interesting information one can deduce from the stability bounds (4.1) concerns dS stationary points. For example they could be of potential interest for achieving inflation. Note nevertheless that there will be at least one direction in field space along which $|V''|/V \sim 1$, implying that the conditions for slow-roll inflation are never satisfied.

In more general situations of $\mathcal{N}=2$ supergravity theories involving both vector and hypermultiplets, as well as non-Abelian gauging, some examples of models giving rise to dS spaces are known to exist [8]. It is clear that an analysis of the same type as the one presented here for these more general situations would also be very valuable, as it could provide some insights on what are the really necessary ingredients to construct models admitting a stable dS vacuum [7]. For instance, it is obvious that non-Abelian gaugings help, since then a new positive-definite term arises in the scalar potential. But even for Abelian gaugings, combining vector multiplets with hypermultiplets may be sufficient to be able to avoid tachyons, since in that case the scalar manifold is the product of a quaternionic-Kähler and special-Kähler manifolds, which as a whole can have a lower sectional curvature than any of its two components. Of course, even after having understood more precisely the conditions for achieving dS vacua within $\mathcal{N}=2$ supergravity effective theories, another interesting question would be whether these can be realized in string theory.

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[17] Under certain (restrictive) circumstances, it is possible to consistently truncate an $\mathcal{N}=2$ theory with n hypermultiplets down to an $\mathcal{N}=1$ theory with n' chiral multiplets [19]. At the geometrical level, this truncation involves the restriction to a Kähler-Hodge submanifold of the original quaternionic-Kähler manifold. Even though the curvature of the Kähler submanifold is arbitrary, the superpotential and thus the sGoldstino directions are more constraint than in generic $\mathcal{N}=1$ theories. It would be interesting to study in more detail the stability conditions in this case.

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A. Supertrace sum rule on the masses

In this appendix we report some details on the computation of the supertrace of the square mass operator for all the fields. This quantity is of some interest, since it controls the leading quadratic divergences arising at the one loop level when supersymmetry is spontaneously broken, at least in the case of a flat Minkowski space with vanishing cosmological constant. We will first shortly review the know case of $\mathcal{N}=1$ theories and then present the same computation for $\mathcal{N}=2$ models.

A.1 $\mathcal{N}=1$ theories with chiral multiplets

Using the expressions given in section 2.1 for the mass matrices of the various fields, one finds that at a generic stationary point with any allowed cosmological constant:

$$\begin{aligned} \mathrm{tr}[m_0^2] &= 2\nabla_i N_k \nabla^i \bar{N}^k - 2R_{i\bar{\jmath}}\bar{N}^i N^{\bar{\jmath}} + 2(n-1)\bar{N}^k N_k - 4n|L|^2\,, && \text{(A.1)} \\ \mathrm{tr}[m_{1/2}^2] &= \nabla_i N_k \nabla^i \bar{N}^k - \frac{8}{3}\bar{N}^k N_k + \frac{4}{9}(\bar{N}^k N_k)^2 |L|^{-2}\,, && \text{(A.2)} \\ \mathrm{tr}[m_{3/2}^2] &= |L|^2\,. && \text{(A.3)} \end{aligned}$$

It follows that: [21]

$$\begin{aligned} \mathrm{str}[m^2] &= \mathrm{tr}[m_0^2] - 2\,\mathrm{tr}[m_{1/2}^2] - 4\,\mathrm{tr}[m_{3/2}^2] \\ &= 2(n-1)m_{3/2}^2 + 2R_{i\bar{\jmath}}\bar{N}^i N^{\bar{\jmath}} + 2(n-1)V - \frac{8}{9}V^2|m_{3/2}|^{-2}\,. \end{aligned} \tag{A.4}$$

In terms of $\gamma = V/(3|m_{3/2}|^2)$, this finally gives:

$$\mathrm{str}[m^2] = \big[2(n-1) + 6(n-1)\gamma - 8\gamma^2\big]|m_{3/2}|^2 + 2R_{i\bar{\jmath}}\bar{N}^i N^{\bar{\jmath}}\,. \tag{A.5}$$

Note that for Kähler manifolds that happen to be also Einstein spaces, with a Ricci tensor of the form

$$R_{i\bar{\jmath}} = r g_{i\bar{\jmath}}\,, \tag{A.6}$$

the formula simplifies as follows:

$$\mathrm{str}[m^2] = \big[2(n-1+3r) + 6(n-1+r)\gamma - 8\gamma^2\big]|m_{3/2}|^2\,. \tag{A.7}$$

Note finally that for supersymmetric vacua with $\gamma = -1$ one finds:

$$\mathrm{str}[m^2] = -4(n+1)|m_{3/2}|^2\,. \tag{A.8}$$

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A.2 $\mathcal{N}=2$ theories with hypermultiplets

Using the expressions derived in section 3.1 for the mass matrices of the various fields, as well as eq. (3.8), one can compute the traces of the square mass for each field at a generic stationary point of the scalar potential. After some algebra, and repeated use of the various identities listed at the beginning of section 3, we find the following results:

$$\begin{aligned}
\mathrm{tr}[m_0^2] &= 4\nabla^u k^v \nabla_u k_v + 4(2n-5)k^u k_u - 12n P^x P^x \,, && \text{(A.9)}\\
\mathrm{tr}[m_{1/2}^2] &= 2\nabla^u k^v \nabla_u k_v - 16 k^u k_u - 2n P^x P^x + \frac{128}{9}(k^u k_u)^2 (P^x P^x)^{-1} \,, && \text{(A.10)}\\
\mathrm{tr}[m_1^2] &= 4k^u k_u \,, && \text{(A.11)}\\
\mathrm{tr}[m_{3/2}^2] &= 2P^x P^x \,. && \text{(A.12)}
\end{aligned}$$

Using these result, the supertrace is found to be:

$$\begin{aligned}
\mathrm{str}[m^2] &= \mathrm{tr}[m_0^2] - 2\,\mathrm{tr}[m_{1/2}^2] + 3\,\mathrm{tr}[m_1^2] - 4\,\mathrm{tr}[m_{3/2}^2] \\
&= -\big(2n+6\big)\,|m_{3/2}|^2 + \left(2n - \frac{14}{3}\right) V - \frac{16}{9}\, V^2 |m_{3/2}|^{-2} \,.
\end{aligned} \tag{A.13}$$

In terms of $\gamma = V/(3|m_{3/2}|^2)$, this finally reads:

$$\mathrm{str}[m^2] = \big[-\big(2n+6\big) + \big(6n-14\big)\gamma - 16\gamma^2\big]|m_{3/2}|^2 \,. \tag{A.14}$$

Note that for supersymmetric vacua with $\gamma = -1$ one finds:

$$\mathrm{str}[m^2] = -8(n+1)\,|m_{3/2}|^2 \,. \tag{A.15}$$

B. Curvature conventions

In this appendix, we summarize our conventions for the curvature tensor and the sectional curvature, first for generic real Riemann manifolds and then for complex Kähler manifolds.

B.1 Riemann manifolds

For the geometry of a generic real Riemann manifold, we use the following conventions. Denoting the components of the metric with g_{uv}, the Christoffel connection is $\Gamma^k_{uv} = \frac{1}{2} g^{kl}\big(\partial_u g_{vl} + \partial_v g_{ul} - \partial_l g_{uv}\big)$. The Riemann tensor is defined as

$$R^u_{\ vkl} = \partial_k \Gamma^u_{vl} - \partial_l \Gamma^u_{vk} + \Gamma^i_{ks}\Gamma^s_{jl} - \Gamma^i_{ls}\Gamma^s_{jk} \,. \tag{B.1}$$

The Ricci curvature tensor is then:

$$R_{ij} = R^s_{\ isj} \,, \tag{B.2}$$

and finally the scalar curvature is given by:

$$R = R^s_{\ s} \,. \tag{B.3}$$

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The ordinary covariant derivatives on vectors is defined as $\mathcal{D}_u V_v = \partial_u V_v - \Gamma^s_{uv} V_s$, and the commutator of two of them gives:

$$\big[\mathcal{D}_u, \mathcal{D}_v\big] V_k = R_{uvk}{}^l V_l \,. \tag{B.4}$$

The sectional curvature in a plane defined by two orthogonal vectors A_u and B_v, with $A^u B_u = 0$, is finally defined as:

$$R(A,B) = \frac{R_{uvkl} A^u B^v A^k B^l}{A^r A_r \, B^s B_s} \,. \tag{B.5}$$

B.2 Kähler manifolds

For complex Kähler manifolds admitting a globally-defined complex structure J_{uv}, it is convenient to switch to complex coordinates in which this is block diagonal with values $\pm i$. The Hermitian metric has non-vanishing components $g_{i\bar{\jmath}}$ and $g_{\bar{\imath}j}$, and satisfies the conditions $\partial_i g_{j\bar{k}} = \partial_j g_{i\bar{k}}$ and $\partial_{\bar{\imath}} g_{\bar{\jmath}k} = \partial_{\bar{\jmath}} g_{\bar{\imath}k}$. It follows then that the non-vanishing components of the Christoffel connection are $\Gamma^k_{ij} = g^{k\bar{l}} \partial_i g_{j\bar{l}}$ and $\Gamma^{\bar{k}}_{\bar{\imath}\bar{\jmath}} = g^{\bar{k}l} \partial_{\bar{\imath}} g_{\bar{\jmath}l}$. The non-vanishing components of the Riemann tensor are then:

$$R_{i\bar{\jmath}k\bar{l}} = \partial_i \partial_{\bar{\jmath}} g_{k\bar{l}} + g^{\bar{r}s} \partial_i g_{k\bar{r}} \partial_{\bar{\jmath}} g_{\bar{l}s} \,, \tag{B.6}$$

$$R_{\bar{\imath}jk\bar{l}} = -R_{j\bar{\imath}k\bar{l}} \,, \quad R_{i\bar{\jmath}\bar{k}l} = -R_{i\bar{\jmath}l\bar{k}} \,, \quad R_{\bar{\imath}j\bar{k}l} = R_{j\bar{\imath}l\bar{k}} \,. \tag{B.7}$$

The Riemann tensor has in this case the additional property of being symmetric under the exchange of indices of the same holomophic or antiholomorphic type: $R_{i\bar{\jmath}k\bar{l}} = R_{k\bar{\jmath}i\bar{l}} = R_{i\bar{l}k\bar{\jmath}} = R_{k\bar{l}i\bar{\jmath}}$. The Ricci curvature tensor has then as only non-vanishing components

$$R_{i\bar{\jmath}} = -g^{r\bar{s}} R_{r\bar{s}i\bar{\jmath}} \,, \qquad R_{\bar{\imath}j} = R_{j\bar{\imath}} \,. \tag{B.8}$$

Finally, the scalar curvature is given by:

$$R = 2 g^{r\bar{s}} R_{r\bar{s}} \,. \tag{B.9}$$

The ordinary covariant derivatives on holomorphic vectors (similar formulae hold for antiholomorphic vectors) read $\mathcal{D}_i V_j = \partial_i V_j - \Gamma^s_{ij} V_s$ and $\mathcal{D}_{\bar{\imath}} V_j = \partial_{\bar{\imath}} V_j$, and the commutator of two of them gives:

$$\big[\mathcal{D}_i, \mathcal{D}_{\bar{\jmath}}\big] V_k = R_{i\bar{\jmath}k}{}^l V_l \,. \tag{B.10}$$

The holomorphic sectional curvature in a plane defined by a vector and its conjugate under the complex structure, defining in complex coordinates a holomorphic vector Z_i and its antiholomorphic counterpart $Z_{\bar{\imath}}$, is finally given by:

$$R(Z) = -\frac{R_{i\bar{\jmath}k\bar{l}} Z^i Z^{\bar{\jmath}} Z^k Z^{\bar{l}}}{(Z^p Z_p)^2} \,. \tag{B.11}$$

References

[1] M. Gomez-Reino and C.A. Scrucca, *Locally stable non-supersymmetric Minkowski vacua in supergravity*, *JHEP* **05** (2006) 015 [hep-th/0602246]; *Constraints for the existence of flat and stable non-supersymmetric vacua in supergravity*, *JHEP* **09** (2006) 008 [hep-th/0606273]; *Metastable supergravity vacua with F and D supersymmetry breaking*, *JHEP* **08** (2007) 091 [arXiv:0706.2785].

[2] L. Covi et al., *de Sitter vacua in no-scale supergravities and Calabi-Yau string models*, *JHEP* **06** (2008) 057 [arXiv:0804.1073]; *Constraints on modular inflation in supergravity and string theory*, *JHEP* **08** (2008) 055 [arXiv:0805.3290].

[3] F. Denef and M.R. Douglas, *Distributions of nonsupersymmetric flux vacua*, *JHEP* **03** (2005) 061 [hep-th/0411183].

[4] P. Breitenlohner and D.Z. Freedman, *Stability in gauged extended supergravity*, *Ann. Phys. (NY)* **144** (1982) 249.

[5] B. de Wit and A. Van Proeyen, *Potentials and symmetries of general gauged $N = 2$ supergravity: Yang-Mills models*, *Nucl. Phys.* **B 245** (1984) 89.

[6] E. Cremmer et al., *Vector multiplets coupled to $N = 2$ supergravity: superHiggs effect, flat potentials and geometric structure*, *Nucl. Phys.* **B 250** (1985) 385.

[7] G. Dall'Agata, M. Gomez-Reino, J. Louis and C.A. Scrucca, work in progress.

[8] P. Fré, M. Trigiante and A. Van Proeyen, *Stable de Sitter vacua from $N = 2$ supergravity*, *Class. and Quant. Grav.* **19** (2002) 4167 [hep-th/0205119].

[9] M. Günaydin and M. Zagermann, *The vacua of 5D, $N = 2$ gauged Yang-Mills/Einstein/tensor supergravity: abelian case*, *Phys. Rev.* **D 62** (2000) 044028 [hep-th/0002228];
B. Cosemans and G. Smet, *Stable de Sitter vacua in $N = 2$, $D = 5$ supergravity*, *Class. and Quant. Grav.* **22** (2005) 2359 [hep-th/0502202];
O. Ogetbil, *Stable de Sitter vacua in 4 dimensional supergravity originating from 5 dimensions*, *Phys. Rev.* **D 78** (2008) 105001 [arXiv:0809.0544].

[10] E. Cremmer et al., *Spontaneous symmetry breaking and Higgs effect in supergravity without cosmological constant*, *Nucl. Phys.* **B 147** (1979) 105;
E. Witten and J. Bagger, *Quantization of Newton's constant in certain supergravity theories*, *Phys. Lett.* **B 115** (1982) 202;
E. Cremmer, S. Ferrara, L. Girardello and A. Van Proeyen, *Coupling supersymmetric Yang-Mills theories to supergravity*, *Phys. Lett.* **B 116** (1982) 231;
E. Cremmer, S. Ferrara, L. Girardello and A. Van Proeyen, *Yang-Mills theories with local supersymmetry: lagrangian, transformation laws and superHiggs effect*, *Nucl. Phys.* **B 212** (1983) 413.

[11] A. Achucarro, S. Hardeman and K. Sousa, *F-term uplifting and the supersymmetric integration of heavy moduli*, *JHEP* **11** (2008) 003 [arXiv:0809.1441]; *Consistent decoupling of heavy scalars and moduli in $N = 1$ supergravity*, *Phys. Rev.* **D 78** (2008) 101901 [arXiv:0806.4364];
A. Achucarro and K. Sousa, *F-term uplifting and moduli stabilization consistent with Kähler invariance*, arXiv:0712.3460.

[12] J. Bagger and E. Witten, *Matter couplings in $N = 2$ supergravity*, *Nucl. Phys.* **B 222** (1983) 1;
B. de Wit, P.G. Lauwers and A. Van Proeyen, *Lagrangians of $N = 2$ supergravity-matter systems*, *Nucl. Phys.* **B 255** (1985) 569;
R. D'Auria, S. Ferrara and P. Fré, *Special and quaternionic isometries: general couplings in $N = 2$ supergravity and the scalar potential*, *Nucl. Phys.* **B 359** (1991) 705;
L. Andrianopoli et al., *General matter coupled $N = 2$ supergravity*, *Nucl. Phys.* **B 476** (1996) 397 [hep-th/9603004].

[13] L. Andrianopoli et al., *$N = 2$ supergravity and $N = 2$ super Yang-Mills theory on general scalar manifolds: symplectic covariance, gaugings and the momentum map*, *J. Geom. Phys.* **23** (1997) 111 [hep-th/9605032].

[14] R. D'Auria and S. Ferrara, *On fermion masses, gradient flows and potential in supersymmetric theories*, *JHEP* **05** (2001) 034 [hep-th/0103153].

[15] D.V. Alekseevsky, V. Cortes, C. Devchand and A. Van Proeyen, *Flows on quaternionic-Kähler and very special real manifolds*, *Commun. Math. Phys.* **238** (2003) 525 [hep-th/0109094].

[16] J. Louis and A. Micu, *Type II theories compactified on Calabi-Yau threefolds in the presence of background fluxes*, *Nucl. Phys.* **B 635** (2002) 395 [hep-th/0202168];
G. Dall'Agata, R. D'Auria, L. Sommovigo and S. Vaula, *$D = 4$, $N = 2$ gauged supergravity in the presence of tensor multiplets*, *Nucl. Phys.* **B 682** (2004) 243 [hep-th/0312210];
B. de Wit, H. Samtleben and M. Trigiante, *Magnetic charges in local field theory*, *JHEP* **09** (2005) 016 [hep-th/0507289].

[17] G. Dall'Agata and A. Van Proeyen, unpublished.

[18] M. Davidse, F. Saueressig, U. Theis and S. Vandoren, *Membrane instantons and de Sitter vacua*, *JHEP* **09** (2005) 065 [hep-th/0506097].

[19] L. Andrianopoli, R. D'Auria and S. Ferrara, *Consistent reduction of $N = 2 \to N = 1$ four dimensional supergravity coupled to matter*, *Nucl. Phys.* **B 628** (2002) 387 [hep-th/0112192];
Supersymmetry reduction of N-extended supergravities in four dimensions, *JHEP* **03** (2002) 025 [hep-th/0110277].

[20] R. Kallosh, A.D. Linde, S. Prokushkin and M. Shmakova, *Gauged supergravities, de Sitter space and cosmology*, *Phys. Rev.* **D 65** (2002) 105016 [hep-th/0110089];
M. de Roo, D.B. Westra and S. Panda, *De Sitter solutions in $N = 4$ matter coupled supergravity*, *JHEP* **02** (2003) 003 [hep-th/0212216];
M. de Roo, D.B. Westra, S. Panda and M. Trigiante, *Potential and mass-matrix in gauged $N = 4$ supergravity*, *JHEP* **11** (2003) 022 [hep-th/0310187].

[21] M.T. Grisaru, M. Roček and A. Karlhede, *The superHiggs effect in superspace*, *Phys. Lett.* **B 120** (1983) 110.